8

Mathematics

Quarter 1 – Module 16
Solving Systems of Linear Equations in Two Variables

ALTERNATIVE DELIVERY MODE

ADM

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Mathematics – Grade 8
Alternative Delivery Mode
Quarter 1 – Module 16 Solving Systems of Linear Equations in Two Variables
First Edition, 2020

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Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer: Melissa R. Cabarles

Language Editor: Merjorie G. Dalagan

Content Evaluator: Hezl M. Evangelio

Layout Evaluator: Jake D. Fraga

Reviewers: Rhea J. Yparraguirre, Nilda A. Mendiola, Manuel L. Limjoco, Jr., Cris Gerom C. Arguilles, Melba G. Lumangcas, Analyn L. Avila, Thelma D. Ramos

Illustrator: Melissa R. Cabarles

Layout Artist: Melissa R. Cabarles

Management Team: Francis Cesar B. Bringas
Isidro M. Biol, Jr.
Maripaz F. Magno
Josephine Chonie M. Obseñares
Josita B. Carmen
Celsa A. Casa
Regina Euann A. Puerto
Bryan L. Arreo
Elnie Anthony P. Barcena
Leopardo P. Cortes, Jr.

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Department of Education – Caraga Region

Office Address: Learning Resource Management Section (LRMS)
J.P. Rosales Avenue, Butuan City, Philippines 8600
Tel. No./Telefax No.: (085) 342-8207 / (085) 342-5969
E-mail Address: caraga@deped.gov.ph

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Mathematics

Quarter 1 – Module 16
Solving Systems of Linear Equations in Two Variables

Introductory Message

For the facilitator:

Welcome to the Mathematics 8 Alternative Delivery Mode (ADM) Module on Solving Systems of Linear Equations in Two Variables!

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:

Notes to the Teacher

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the Mathematics 8 Alternative Delivery Mode (ADM) Module on Solving Systems of Linear Equations in Two Variables!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Need to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
What I Know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
What's In	This is a brief drill or review to help you link the current lesson with the previous one.
What's New	In this portion, the new lesson will be introduced to you in various ways; a story, a song, a poem, a problem opener, an activity or a situation.
What is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
What I Have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned.
Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don’t forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to help you solve problems involving systems of linear equations in two variables using graphical and algebraic (substitution and elimination) methods. Throughout this module, you will be provided with varied activities to process your knowledge and skills acquired, deepen and transfer your understanding of the algebraic methods of solving systems of linear equations in two variables. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains:

Lesson 1: Solving Problems Involving Systems of Linear Equation in Two Variables

After going through this module, you are expected to:

1. solve systems of linear equations in two variables by substitution and elimination methods;
2. solve problems involving systems of linear equations in two variables by graphing, substitution and elimination; and
3. determine the most efficient method in solving system of linear equations in two variables.

What I Know

Choose the letter of the correct answer. Write the letter of the correct answer on a separate sheet of paper.

1. What do you call the process of adding the equations to eliminate either x or y from the system of linear equations?

   A. cancellation
   B. elimination
   C. graphing
   D. substitution

2. Determine which coordinates satisfy the system $\begin{cases} 5x + y = 14 \\ 4x - y = 4 \end{cases}$.

   A. $(-4,\ 6)$
   B. $(4, -2)$
   C. $(2, 4)$
   D. $(6, 4)$

3. Is the ordered pair $(2, 0)$ a solution to the system $\begin{cases} x + 2y = 2 \\ 2x - y = 4 \end{cases}$?

   A. No, $(2, 0)$ is definitely not a solution.
   B. Yes, (2, 0) completely satisfies the system**.**
   C. The given is not a system of linear equation.
   D. There is no enough data given to solve the system.

4. What do you call the process of solving one of the equations for one variable and replacing the resulting expression to the other equation to solve for the other variable without changing the value of the original expression?

   A. elimination
   B. graphing
   C. substitution
   D. transformation

5. What is the first step to solve this system of linear equations $\begin{cases} 5x - 3y = -2 \\ 4x + 3y = 20 \end{cases}$?

   A. Add the 2 equations.
   B. Subtract the 2 equations.
   C. Multiply the second equation by 3.
   D. Multiply the first equation by $\frac{1}{5}$.

6. What do you call a system of equations that would give a true statement after performing addition or subtraction to eliminate a variable?

   A. consistent
   B. dependent
   C. inconsistent
   D. independent

7. What is the value of y in the system of linear equations $\begin{cases} x + 3y = 2 \\ 2x + 2y = 8 \end{cases}$?

   A. -5
   B. -1
   C. 1
   D. 5

8. By substitution method in the system of linear equations $\begin{cases} x + 2y = 6 \\ x + 2y = 12 \end{cases}$, what classification is it?

A. consistent
B. dependent
C. inconsistent
D. independent

9. Using the elimination method in the system of linear equation $\begin{cases} 3x - 6y = 12 \\ 6x - 12y = 24 \end{cases}$, how will you describe this system of equation?

A. consistent
B. dependent
C. inconsistent
D. independent

10. Which ordered pair is the solution to this system $\begin{cases} 5x + y = 3 \\ 2x - y = 4 \end{cases}$?

A. $(1, -2)$
B. $(2, -1)$
C. $(2,\ 7)$
D. $(1, -7)$

11. Gio was asked by his teacher to solve the system $\begin{cases} x + 2y = 5 \\ y = x - 4 \end{cases}$. He then presented his solution in the following manner:

	$x + 2y$	$=$	5	*Given (Equation 1)*
Step 1.	$x + 2x - 4$	$=$	5	*Substitute for y: $y = x - 4$*
Step 2.	$3x - 4$	$=$	5	*Combine like terms*
Step 3.	$3x - 4 + 4$	$=$	$5 + 4$	*Add 4 to both sides*
Step 4.	$3x$	$=$	9	*Simplify*
Step 5.	$\frac{1}{3}(3x)$	$=$	$\frac{1}{3}(9)$	*Multiply both sides by $\frac{1}{3}$*
Step 6.	x	$=$	3	*Solve for x*

What was Gio's computational error?

A. In Step 1, Gio wrongly substituted the value of y. It should have been $\boldsymbol{x} + 2(x - 4) = 5$.
B. In Step 3, Gio wrongly added 4 to both sides of the equation. It should have been $3x - 4 - 4 = 5 - 4$.
C. In Step 5, Gio wrongly multiplied $\frac{1}{3}$ to both sides of the equation. It should have been $3(3x) = 3\ (9)$.
D. Gio did not commit any computational error. All the steps are performed logically and accurately.

12. Below are the steps in solving problems involving systems of linear equations in two variables. Which of the following is arranged in a chronological order?

    I. Read and understand the problem.
    II. Check to see if all information is used correctly and that the answer makes sense
    III. Translate the facts into a system of linear equations.
    IV. Solve the system of equations using one of the methods in solving system of linear equations.

    A. I, II, III, and IV
    B. I, II, IV and III
    C. II, I, III and IV
    D. I, III, IV and II

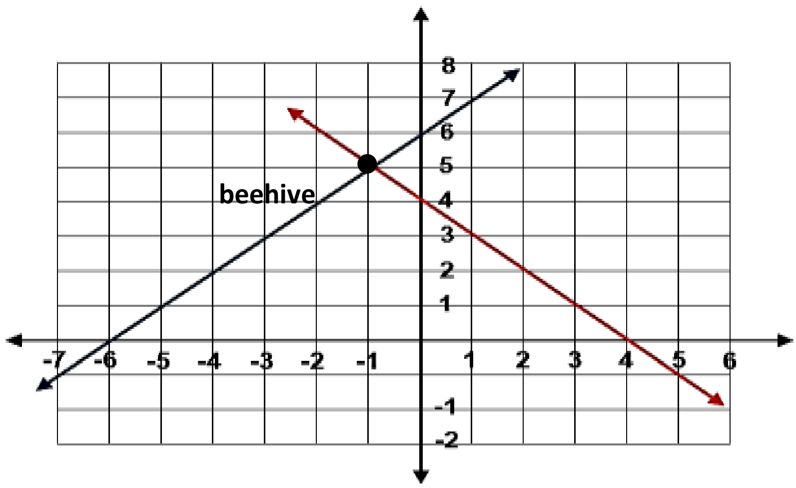
13. Jessie has 2 apples and 3 oranges with a total cost of ₱105.00 while his friend has 1 apple and 4 oranges cost ₱90.00. Which of the following steps would be the best way to begin with in finding the cost of an apple and the cost of an orange?

$$2x + 3y = 105 \quad \text{(equation 1)}$$
$$x + 4y = 90 \quad \text{(equation 2)}$$

    A. Multiply equation 2 by -2.
    B. Multiply equation 1 by 4 and 2.
    C. Add equation 1 and 2.
    D. Multiply equation 2 by 2 and add.

14. A farmer is tracking two wild honey bees in his field. He maps the first bee's path to the hive on the line x + y = 4. The second bee's path follows the line –x + y = 6. Their paths cross at the hive. At what coordinates will the farmer find the hive?

    A. (5, -1)
    B. (1, 5)
    C. (-1, 5)
    D. (1, 5)



15. Four years ago, Luna was 6 times as old as her cousin, six years ago, her age was 2 years more than eight times her cousin's age. How old is Luna?

    A. 20 years old
    B. 25 years old
    C. 30 years old
    D. 40 years old

Lesson 1	Solving Problems Involving Systems of Linear Equations in Two Variables

In the previous module, you learned about solving systems of linear equations in two variables by graphical method. How did you find the lesson? Was it easy to determine the ordered pair that satisfies both equation? Have you ever wondered if there are other ways of finding the solutions of the system of equations other than graphing?

Let us start this lesson by reactivating your knowledge in solving linear equations for a given variable.

What's In

Transform Me in Terms of the Other

Directions: Express each equation in terms of the indicated variable then answer the questions that follow. The first item is done for you.

Original Equation	Transformed Equation
1. $x + y = 2$	$x = -y + 2$
2. $x - y = \frac{2}{3}$	$x =$ ______________________
3. $3x + 2y = 6$	$x =$ ______________________
4. $\frac{1}{4}x + y = 2$	$y =$ ______________________
5. $4x - 3y = -33$	$y =$ ______________________

Questions:

1. Was it easy to solve for one variable in terms of the other?
2. In Item No. 4, was it easy to solve for y in terms of x? How would it be different if you were asked to solve for x in terms of y?
3. If you will graph the equation in Item No. 5 in one Cartesian Plane, would it be easy for you to locate the points? Why or why not?

What's New

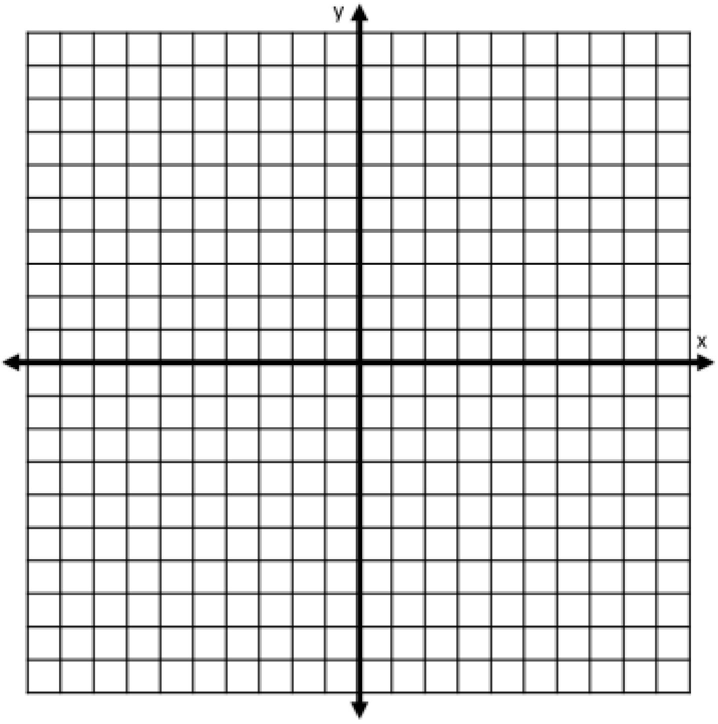
Find Me

Graph the system of equations in items 2 and 5 in the previous activity. Identify the point of intersection, check your solutions and answer the questions that follow.

$$\begin{cases} x - y = \frac{2}{3} \\ 4x - 3y = -33 \end{cases}$$

Questions:

1. How did you find the activity? Was it easy to locate the points of the equation $x - y = \frac{2}{3}$ using the slope-intercept form? How about the equation $4x - 3y = -33$?
2. At what point does the graph of $x - y = \frac{2}{3}$ and $4x - 3y = -33$ intersect?
3. What challenges did you encounter in solving the system by graphing?
4. Can there be other ways to solve the system of equations to identify the accurate point of intersection of the graph or the solution?



Graphing systems of linear equations in two variables can be easily done if the points on each line are integers, but it can be challenging if the coordinates are in fractional form. In some cases, it may still be challenging to determine the accurate coordinates even if the points of intersection are integers especially if the scale of your graph is too small and not properly labeled. Hence, an algebraic method will be introduced.

What is It

Solving Systems of Linear Equations in Two Variables by Substitution

There are several methods for solving system of linear equations other than graphing. One of these is the **substitution** method. When using the **substitution method** we use the fact that if two expressions y and x are of equal value $x = y$, then x may replace y or vice versa in another expression without changing the value of the expression. (Mathplanet.com)

Below are the illustrative examples to help you solve systems of linear equations in two variables using substitution method.

Illustrative example1

Solve the system by substitution: $\begin{cases} x + y = 3 & \quad Equation\ 1 \\ y = x - 1 & \quad Equation\ 2 \end{cases}$

Step 1. Solve an equation for one variable.

$\boldsymbol{y}$	$=$	$\boldsymbol{x - 1}$	*Equation* 2 *is already solved for* y *in terms of x*

Step 2. Substitute that expression for y in the other linear equation to find x.

$\boldsymbol{x + y}$	$=$	3	*Given (Equation 1)*
$x + (x - 1)$	$=$	3	*Equation 2 is already solved for y*
$(x + x) - 1$	$=$	3	*Associative Property of Addition*
$2\,x - 1$	$=$	3	*Combine like terms*
$2x - 1 + 1$	$=$	$3 + 1$	*Add (+1) to both sides of the equation Addition Property of Equality*
$2x$	$=$	4	*Additive Inverse*
$\frac{1}{2}(2x)$	$=$	$(4)\,\frac{1}{2}$	*Multiply both sides* $\frac{1}{2}$ *by Multiplication Property of Equality*
$\boldsymbol{x}$	$=$	$\boldsymbol{2}$	*By simplification.*

Step 3. To find the value of y, substitute the value of x obtained in Step 2 in either of the original equations. For this example we use equation since y is already expressed in terms of x:

$\boldsymbol{y}$	$=$	$\boldsymbol{x - 1}$	*Given (Equation 2)*
y	$=$	$2 - 1$	Substitute 2 to x in the equation
$\boxed{y \quad = \quad 1}$			*By simplification.*

Therefore, the ordered pair obtained is $(2, 1)$.

Step 4. Check the solution. Substitute the value of x and y obtained in $Step\ 3$.

For equation 1:

$$\begin{aligned} x+y &= 3 \\ 2+1 &= 3 \\ 3 &= 3\ ✓ \end{aligned}$$

For equation 2:

$$\begin{aligned} y &= x-1 \\ 1 &= 2-1 \\ 1 &= 1\ ✓ \end{aligned}$$

Both values of x and y satisfy the equations; therefore, the solution to the system is $x = 2$ and $y = 1$ or may be written as $(2, 1)$.

> Recall that in your previous lesson, system of linear equations was classified as consistent and independent, consistent and dependent, or inconsistent. Using substitution method, we can also do the same. The system presented in Illustrative example 1 is **consistent and independent system** since it **has exactly one solution**.

Illustrative example 2: Solve the system $\begin{cases} y = 2x + 5 \\ 2x = y + 3 \end{cases}$

Step 1. Solve an equation for one variable.

$\boldsymbol{y} \quad = \quad \mathbf{2}\boldsymbol{x} + \mathbf{5}$ *Equation 1 is already solved for y in terms of x*

Step 2. Substitute the expression for y in the other linear equation to find x.

$2x$	$=$	$y + 3$	*Given (Equation 2)*
$2x$	$=$	$(2x + 5) + 3$	*Substitute the value of y obtained in Step 1*
$2x$	$=$	$2x + 8$	*Combine like terms*
$2x - 2x$	$=$	$2x + 8 - 2x$	*Add $(-2x)$ to both sides by Addition Property of equality*
$\mathbf{0}$	$=$	$\mathbf{8}$	***FALSE***

> The result $\mathbf{0} = \mathbf{8}$ in the previous page is a false statement. This means that for any values of x and y, there is no ordered (x, y) that would satisfy the system of equation. Hence, it has no solution. Thus, the system is **inconsistent.**

Illustrative Example 3:

Solve the system $\begin{cases} x + y = 3 \\ 3x + 3y = 9 \end{cases}$

Step 1. Solve an equation for one variable.

$x + y = 3 \quad \Rightarrow \quad y = -x + 3$ *Solve for y in terms of x*

Step 2. Substitute the solution in Step 1 for y in equation 2 and solve for x

$3x + 3y$	$=$	9	*Given (Equation 2)*
$3x + 3(-x + 3)$	$=$	9	*Substitute the value of y obtained in Step 1*
$3x - 3x + 9$	$=$	9	*Distributive Property*
$\mathbf{9}$	$=$	$\mathbf{9}$	***TRUE***

Notice that in Step 2, the resulting equation $9 = 9$ is a true statement. The fact that the statement is true for any value of x and y, tells us that the solution set of the given system is an infinite set where all the ordered pairs lie on the same line. Hence the given system of equations is **consistent and dependent**.

Solving Systems of Linear Equations in Two Variables by Elimination

Another method of solving systems of linear equations in two variables is the **elimination method**. The objective of this method is to eliminate one of the variables to find the other variable.

This method requires us to add two equations or subtract one from another in order to eliminate either x or y. Oftentimes, one may not proceed with the addition directly without first multiplying either the first or second equation by some value. (Mathplanet.com)

For your guide, consider the examples below.

Illustrative Example 1: Solve the systems of equation by elimination: $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

Step 1. Write the equations in the standard form $Ax + By = C$. If the given equations are already in standard form, proceed to Step 2.

Note that each equation in the system is already in the form $Ax + By = C$, hence, proceed to Step 2.

Step 2. Add or subtract the equations to eliminate one variable, and then solve for the other variable. In this example, note that the $y - term$ has a numerical coefficient that are opposite, that is $1\ and\ -1$. Hence, addition can be used.

$x + y$	$=$	3	*Given (Equation 1)*
$x - y$	$=$	1	*Given (Equation 2)*
$2x + 0$	$=$	4	*Eliminate y by addition*
$2x$	$=$	4	*Additive Identity*
$\frac{1}{2}(2x)$	$=$	$\frac{1}{2}(4)$	*Multiply both sides by $\frac{1}{2}$ by Multiplication Property of Equality*
$\boldsymbol{x}$	$=$	$\mathbf{2}$	*By simplification.*

Step 3. Substitute the value of x in either of the equations and solve for the value of y.

$x + y$	$=$	3	*Given (Equation 1)*
$2 + y$	$=$	3	Substitute x by 2
$2 + (-2) + y$	$=$	$(-2) + 3$	Add (-2) to both sides by Addition Property of Equality
$\boxed{\boldsymbol{y = 1}}$			*By simplification*.

Therefore, the ordered pair obtained is $(2, 1)$.

Step 4. Check the solution. Substitute the value of x and y obtained in $Step\ 3$.

For equation 1:

$$x + y = 3$$
$$2 + 1 = 3$$
$$3 = 3 \checkmark$$

For equation 2:

$$y = x - 1$$
$$1 = 2 - 1$$
$$1 = 1 \checkmark$$

Both values of x and y satisfy the equations; therefore, the solution to the system is $x = 2$ and $y = 1$ or may be written as $(2, 1)$.

Illustrative Example 2: Solve the system $\begin{cases} -5y = -2x + 3 \\ 3x + y = -4 \end{cases}$ by elimination.

Step 1. Write the equations in the standard form $Ax + By = C$.

$-5y = -2x + 3$	$\Longrightarrow$	$2x - 5y = 3$	*Standard form of Equation 1*
		$3x + y = -4$	*Equation 2 is already in standard from*

The system is now $\begin{cases} 2x - 5y = 3 \\ 3x + y = -4 \end{cases}$

Step 2. Add or subtract the equations to eliminate one variable, and then solve for the other variable. To add or subtract, coefficients must be opposites or the same.

$2x - 5y$	$=$	3	*Standard form of Equation 1*
$3x + y$	$=$	-4	*Equation 2*
$5(3x + y)$	$=$	$5(-4)$	Multiply each term in equation 2 by 5 so that the coefficients of y in both Equations 1 and 2 become opposites
$\mathbf{15x + 5y}$	$=$	$\mathbf{-20}$	***Equivalent equation of*** $\mathbf{3x + y = -4}$
$2x - 5y$	$=$	3	*Equation 1*
$15x + 5y$	$=$	-20	*Equivalent of Equation 2*
$17x + 0$	$=$	-17	*Eliminate y by adding the 2 equations*
$17x$	$=$	-17	*Additive Identity*
$\frac{1}{17}(17x)$	$=$	$\frac{1}{17}(-17)$	*Multiply both sides by $\frac{1}{17}$ by Multiplication Property of Equality*
$\boxed{\boldsymbol{x = -1}}$			*By simplification.*

Step 3. Substitute the value of x in either of the equations and solve for the value of y.

$-5y$	$=$	$-2x + 3$	*Given (Equation 1)*
$-5y$	$=$	$-2(-1) + 3$	*Substitute x by -1*
$-5y$	$=$	$2 + 3$	*Simplify*
$-5y$	$=$	5	
$-\frac{1}{5}(-5y)$	$=$	$-\frac{1}{5}(5)$	*Multiply both sides by $-\frac{1}{5}$ by Multiplication Property of Equality*
$\boldsymbol{y}$	$=$	$\boldsymbol{-1}$	*By simplification.*

Therefore, the ordered pair obtained is $(-1, -1)$.

Step 4. Check the solution. Substitute the value of x and y obtained in $Step\ 3$.

For equation 1:

$$\begin{aligned} -5y &= -2x + 3 \\ -5(-1) &= -2(-1) + 3 \\ 5 &= 2 + 3 \\ 5 &= 5 \ \checkmark \end{aligned}$$

For equation 2:

$$\begin{aligned} 3x + y &= -4 \\ 3(-1) + (-1) &= -4 \\ -3 - 1 &= -4 \\ -4 &= -4 \ \checkmark \end{aligned}$$

Both values of x and y satisfy the equations; therefore, the solution to the system is $x = -1$ and $y = -1$ or may be written as $(-1, -1)$.

Illustrative Example 3:

There are equations that contain fractions or parentheses. This should be simplified to the standard form Ax + By = C, then proceed to addition or subtraction.

Solve the system
$$\frac{6}{7}x + 2 = \frac{1}{7}y$$
$$\frac{1}{2}y - \frac{3}{5}x = 4$$

Step 1. Eliminate the fractions by multiplying each side of the equation by a common denominator, then write the simplified equation in standard form.

$\frac{6}{7}x + 2$	$=$	$\frac{1}{7}y$	*Given (Equation 1)*
$7\left(\frac{6}{7}x + 2\right.$	$=$	$\left.\frac{1}{7}y\right)$	*The LCD is 7. Multiply each term of the equation by 7.*
$6x + 14$	$=$	y	*Distributive Property. Simplification.*
$\boldsymbol{6x - y}$	$=$	$\boldsymbol{14}$	***Standard form of Equation 1***
$\frac{1}{2}y - \frac{3}{5}x$	$=$	4	*Equation 2*
$10\left(\frac{1}{2}y - \frac{3}{5}x\right.$	$=$	$\left.4\right)$	*The LCD is 10. Multiply each term by the LCD.*
$5y - 6x$	$=$	40	Distributive Property. Simplification.

$$\mathbf{-6x + 5y = 40} \qquad \textit{\textbf{Standard form of Equation 2}}$$

Hence, the simplified system of equations is $\begin{cases} \mathbf{6x - y = -14} \\ \mathbf{-6x + 5y = 40} \end{cases}$

Step 2. Eliminate one variable by addition or subtraction. To do this, add the equations containing variable with opposite numerical coefficients or subtract the equations with variable with the same numerical coefficients. In this example, note that the numerical coefficients of the $x - terms$ are opposite, that is, $6\ and\ -6$.

$\mathbf{6x - y}$	$=$	$\mathbf{-14}$	*Equation 1*
$\mathbf{-6x + 5y}$	$=$	$\mathbf{40}$	*Equation 2*
$4y$	$=$	26	*Eliminate the variable x by addition.*
$\frac{1}{4}(4y)$	$=$	$\frac{1}{4}(26)$	*Multiply both sides by $\frac{1}{4}$, by Multiplication Property of Equality*
$\mathbf{y}$	$=$	$\mathbf{\frac{26}{4}}$ ***or*** $\mathbf{\frac{13}{2}}$	*By simplification*

Step 3. Substitute the value of y in either of the equations and solve for the value of y.

$\mathbf{6x - y}$	$=$	$\mathbf{-14}$	*Given (Equation 1)*
$6x - \frac{13}{2}$	$=$	-14	Substitute y by $\frac{13}{2}$
$2\left(6x - \frac{13}{2}\right)$	$=$	$2(-14)$	*Multiply both sides by 2.*
$12x - 13$	$=$	-28	*Distributive Property*
$12x - 13 + 13$	$=$	$-28 + 13$	*Add 13 to both sides of the equation by Addition Property of Equality*
$12x$	$=$	-15	*Additive Inverse*
$\frac{1}{12}(12x)$	$=$	$\frac{1}{12}\left(\frac{-15}{12}\right)$	*Multiply both sides by $\frac{1}{12}$ by Multiplication Property of Equality*
$\mathbf{x}$	$=$	$\mathbf{\frac{-15}{12}}$ ***or*** $\mathbf{\frac{-5}{4}}$	*By simplification.*

Therefore, the ordered pair obtained is $(-\mathbf{\frac{5}{4}}, \mathbf{\frac{13}{2}})$

Step 4. Check the solutions.

For equation 1: $\mathbf{6x - y = -14}$			For equation 2: $\mathbf{-6x + 5y = 40}$		
$6\left(-\frac{5}{4}\right) - \frac{13}{2}$	$=$	-14	$-6(-\frac{5}{4}) + 5(\frac{13}{2})$	$=$	40
$-\frac{30}{4} - \frac{13}{2}$	$=$	-14	$\frac{30}{4} + \frac{65}{2}$	$=$	40
$-\frac{30}{4} - \frac{26}{4}$	$=$	-14	$\frac{30}{4} + \frac{130}{4}$	$=$	40
$-\frac{56}{4}$	$=$	-14	$\frac{160}{4}$	$=$	40
-14		-14 ✓	40		40 ✓

Both values satisfy the equation; therefore, the solution to the system is $\boldsymbol{x} = -\frac{5}{4}$ and $\boldsymbol{y} = \frac{13}{2}$ or may be written as $\left(-\frac{5}{4}, \frac{13}{2}\right)$.

Recall that through graphing and substitution methods, it has been shown that systems of equations in two variables can be *consistent and independent, consistent and dependent, or inconsistent*. This can also be possible using the elimination method. Notice that each system of equations presented in Examples 1, 2, and 3 has a unique solution. This means that those systems of equations are *consistent and independent.*

Now, let us determine whether the system of equations consistent and dependent or inconsistent using the elimination method. To do this, examine the additional examples below.

Illustrative example 4: Solve the system $\begin{cases} 4x - 2y = 8 \\ 2x - y = 9 \end{cases}$

Step 1. Write the equations in the standard form $Ax + By = C$.

Since equations 1 & 2 are already expressed in the standard form $Ax + By = C$, then the next step could be done.

Step 2. Eliminate one variable by addition or subtraction. Since none of the equations contain variables whose coefficients are the same or opposite, multiply one or both of the given equations by a number or numbers which will make the coefficients of one of the variables the same or opposite in both equations and perform addition or subtraction.

$2x - y$	$=$	9	*Equation 2*
$-2(2x - y$	$=$	$9)$	*Multiply both sides by* -2
$\boldsymbol{-4x + 2y}$	$=$	$\boldsymbol{-18}$	*Distributive Property* *New Equation 2*
$4x - 2y$	$=$	8	*Equation 1*
$-4x + 2y$	$=$	-18	*Equivalent of Equation 2*
0	$=$	-10	*Elimination by Addition*

Notice that $0 = -10$ is a false statement. The fact that statement $0 = -10$ is not true for any values of x and y, it tells us that there is no solution set. Hence the given system of equations is **inconsistent.**

Illustrative example 5: Solve the system $\begin{cases} x - 4y = 12 \\ 2x - 8y = 24 \end{cases}$

Since both equations are already in the standard form $Ax + By = C$, then we can proceed directly to the next step. To eliminate one variable, multiply one or both equations by a number or numbers which will make the coefficients of either x or y the same or opposite in both equations and perform addition or subtraction.

$x - 4y$	$=$	12	*Equation 1*
$-2(x - 4y$	$=$	$12)$	*Multiply both sides by* -2
$\boldsymbol{-2x + 8y}$	$=$	$\boldsymbol{-24}$	*Distributive Property, New Equation 1*

$$\begin{array}{rcl} -2x+8y & = & -24 \\ 2x-8y & = & 24 \\ \hline 0 & = & 0 \end{array}$$

Equivalent of Equation 1
Equation 2
Elimination by Addition

Notice that $0 = 0$ is a true statement. The fact that the statement is true for any value of x and y, tells us that the solution set of the given system is an infinite set with all the ordered pairs lie on the same line. Hence the given system of equations is **dependent**.

Solving Problems Involving Systems of Linear Equations in Two Variables

You have learned that systems of linear equations in two variables can be solved by graphing, substitution and elimination methods. This time, let us apply these methods in solving problems involving real-life scenarios. We can solve these problems by translating them to systems of equations and by using the problem-solving procedures as enumerated below.

Steps in problem-solving: (Polya's Approach)

1. **Understand the problem.** Read the problem carefully and decide which quantities are unknown.
2. **Develop a plan (Translate).** Study the stated facts until you understand their meaning. Then translate the related facts into equations in two variables.
3. **Carry out the plan (Solve).** Use one of the methods for solving systems of equations. State the conclusions clearly. Include unit of measure if applicable.
4. **Looking back.** Check answers directly against the facts of the problems. Write a statement to answer the question being asked in the problem.

Below are the illustrative examples applying the different methods of solving systems of linear equations in two variables you learned in the previous modules.

Illustrative Example 1: (Age Problem)

The sum of Janna age and Jelo's age is 40. Two years ago, Janna was twice as old as Jelo. Find Janna's age now.

Step 1: Understand the problem.

Make sure that you read the question carefully several times. Since we are looking for Janna's age, we will let

$\boldsymbol{x}$ = Janna's age,

$\boldsymbol{y}$ = Jelo's age

Step 2: Devise a plan (translate).

Since we have 2 unknowns, we need to form a system with two equations.

For equation 1: The sum of Janna and Jelo's age is 40. $\Rightarrow$ $\boldsymbol{x + y = 40}$

For equation 2: Two years ago, Janna was twice as old as Jelo.

Janna (two years ago) = 2 times Jelo's age (two years ago)

$x - 2 = 2(y - 2)$ *Simplify. Use distributive Property*

$x - 2 = 2y - 4$ *Combine like terms*

$\boldsymbol{x - 2y = -2}$

Putting the two equations together in a system we get:

Equation 1 $x + y = 40$
Equation 2 $x - 2y = -2$

Step 3: Carry out the plan (solve).

Use one of the methods for solving systems of equations. For this example, we use elimination method. Since the numerical coefficients of x are equal, we can use elimination by subtraction.

$x + y = 40$ Equation 1

$x - 2y = -2$ Equation 2

To find Jelo's age,

$$\begin{array}{rcl} & x + y = & 40 \\ - & x - 2y = & -2 \\ \hline & 3y = & 42 \end{array}$$ *Eliminate x by subtraction & solve for y*

$\frac{1}{3}(3y) = \frac{1}{3}(42)$ *Multiply both sides by $\frac{1}{3}$ by Multiplication Property of Equality*

$\boldsymbol{y = 14}$ *By simplification*

To find Janna's age

$x + y = 40$ *Use equation 1 to find Janna's age*

$x + 14 = 40$ *Substitute the value of y obtained*

$x + 14 - 14 = 40 - 14$ *Add both sides by -14 by Addition Property of Equality*

$\boldsymbol{x = 26}$ *By simplification*

Step 4: Look back (check and interpret).

Check answers directly against the facts of the problems. Substitute the value of x and y to both equations

Sum of Janna's and Jello's age

$$\begin{aligned} x + y &= 40 \\ 26 + 14 &= 40 \\ 40 &= 40 \end{aligned}$$

Two years ago, Janna was twice as old as Jelo

$$\begin{aligned} -x + 2y &= 2 \\ -26 + 2(14) &= 2 \\ -26 + 28 &= 2 \\ 2 &= 2 \end{aligned}$$

Therefore: Janna's age is 26

Illustrative Example 2: (Number Problem)

The sum of two numbers is 14 and their difference is 2. Find the numbers.

Step 1: Understand the problem.

Make sure that you read the question carefully several times. Since we are looking for the two numbers, we will let

x = first number,
y = second number

Step 2: Devise a plan (translate).

Since we have 2 unknowns, we need to form a system with two equations

For equation 1: The sum of two numbers is 14 $\Rightarrow$ $x + y = 14$

For equation 2: Their difference is 2. $\Rightarrow$ $x - y = 2$

Putting the two equations together in a system we get:

$$\begin{cases} x + y = 14 & \text{Equation 1} \\ x - y = 2 & \text{Equation 2} \end{cases}$$

Step 3: Carry out the plan (solve).

Use one of the methods for solving systems of equations. For this example, we use elimination method since the coefficients of variable y are already the same and with opposite signs.

To find the first number:

$x + y$	$=$	14	*Eliminate y by addition to find the value of x*
$\underline{x - y}$	$\underline{=}$	$\underline{2}$	
$2x$	$=$	16	
$\frac{1}{2}(2x)$	$=$	$\frac{1}{2}(16)$	*Multiply both sides by $\frac{1}{2}$ by Multiplication Property of Equality*
$\boldsymbol{x}$	$=$	$\mathbf{8}$	*By simplification.*

To find the second number.

$x + y$	$=$	14	*Use equation 1 to find the second number*
$8 + y$	$=$	14	*Substitute the value of x*
$8 + y - 8$	$=$	$14 - 8$	*Add (-8) to both sides by Addition Property of Equality*
$\boldsymbol{y}$	$=$	$\mathbf{6}$	*By simplification*

Step 4: Look back (check and interpret).

Check answers directly against the facts of the problems. Substitute the value of x and y to both equations.

$$x + y = 14 \qquad\qquad x - y = 2$$
$$8 + 6 = 14 \qquad\qquad 8 - 6 = 2$$
$$14 = 14 \checkmark \qquad\qquad 2 = 2 \checkmark$$

Therefore, the first number is 8, and the second number is 6

Illustrative Example 3: (Break-even Point Problem)

Mrs. Perez is trying to decide between two hotels to be the venue of her daughter's 18^{th} birthday celebration. Both venues are spacious and elegant and can provide LED screen. Hotel ***A*** charges Php12,000.00 for the venue rental, plus an additional Php500.00 per hour for the extended hours used. Hotel ***B*** charges Php10,500.00 for the venue rental, plus Php1,000.00 per hour for the extended hour used. At how many hours will the two hotels charge the same amount of money? If you are to recommend to Mrs. Perez as to which of the two venues shall be the venue of her daughter's 18^{th} birthday celebration, which hotel will you recommend? Why?

Step 1: Understand the problem.

Make sure that you read the question carefully several times. Since we are looking for an ordered pair, we will let

x = number of hours of extended use
y = total cost (rental cost plus the additional charge per hour)

Step 2: Devise a plan (translate).

Since we have 2 unknowns, we need to form a system with two equations

For equation 1 $\quad y = 500x + 12{,}000$

For equation 2 $\quad y = 1{,}000x + 10{,}500$

Step 3: Carry out the plan (solve).

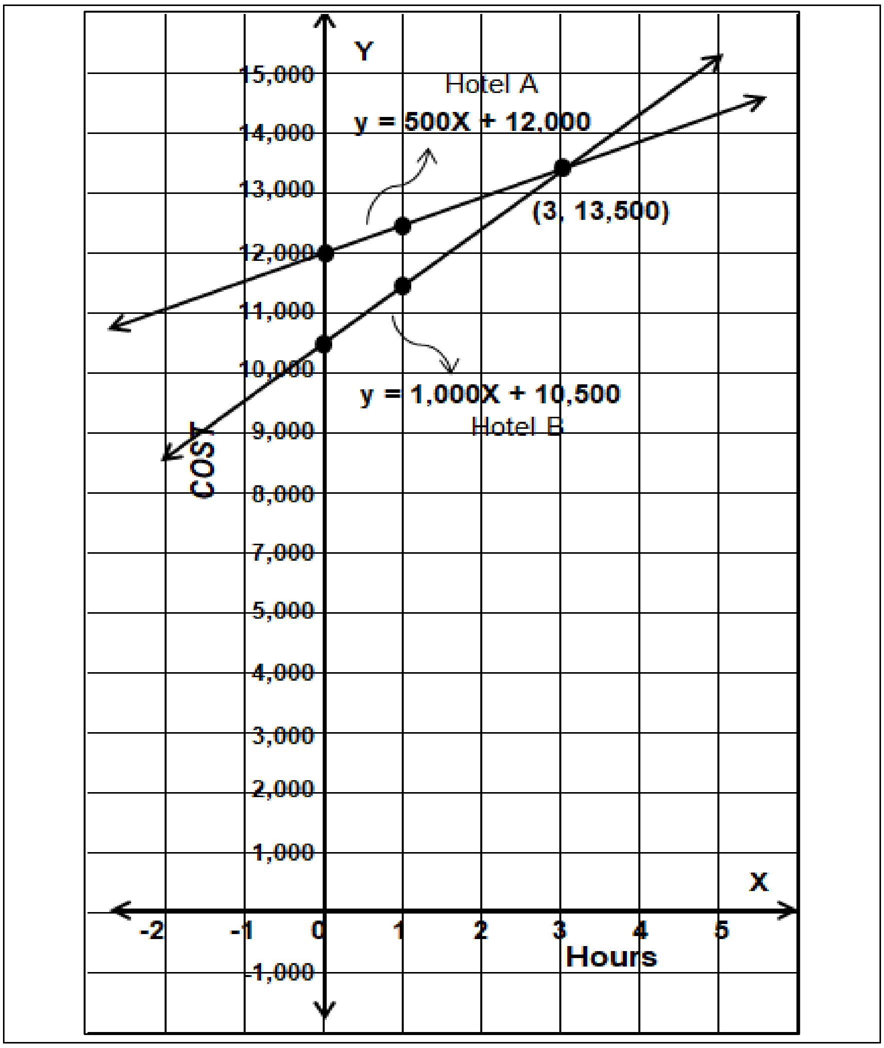
Use one of the methods for solving systems of equations. Since the problem asks us to determine the number of hours where the two hotels charges the same amount of money, we can solve this graphically.

To graph the equations obtained in step 2, simply determine the slopes and y-intercepts of the equations since the two equations obtained in step 2 are already in slope-intercept form.

For eq.1 $y = 500x + 12{,}000$; m = 500; b = 12,000

For eq.2 $y = 1{,}000x + 10{,}500$; m = 1,000; b = 10,500

The point of intersection of the graphs refers to a point where the two hotels charge the same amount of money.



Step 4: Look back (check and interpret).

Check answers directly against the facts of the problems. Substitute the value of x and y to both equations.

$$y = 500x + 12{,}000 \qquad\qquad y = 1000x + 10{,}500$$

$$13{,}500 = 500(3) + 12{,}000 \qquad\qquad 13{,}500 = 1000(3) + 10{,}500$$

$$13{,}500 = 1,500 + 12{,}000 \qquad\qquad 13{,}500 = 3,000 + 10{,}500$$

$$13{,}500 = 13{,}500 \qquad\qquad 13{,}500 = 13{,}500$$

Therefore, the number of hours the two hotels charges the same amount is when the extended hours reach 3 hours at Php13,500.00

To answer Question No. 2, let us find the value of y when the extended number of hours is less than 3 hours and when the extended number of hours is more than 3 hours. Suppose we solve for y in both equations when $x = 2$ and when $x = 4$.

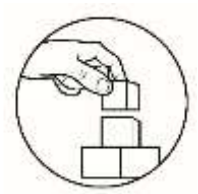
Equation 1: $y = 500x + 12{,}000$ (Hotel A)	Equation 2 : $y = 1000x + 10{,}500$ (Hotel B)
Solve for y, when $x = 2$	Solve for y, when $x = 2$
$y = 500x + 12{,}000$	$y = 1000x + 10{,}500$
$y = 500(2) + 12{,}000$	$y = 1000(2) + 10{,}500$
$y = 1{,}000 + 12{,}000$	$y = 2{,}000 + 10{,}500$
$\boldsymbol{y}$ $\mathbf{13,000}$	$\boldsymbol{y}$ $\mathbf{12,500}$

At $x = 2$, the cost of Hotel A is higher than the cost of Hotel B.

Equation 1: $y = 500x + 12{,}000$ (Hotel A)	Equation 2 : $y = 1000x + 10{,}500$ (Hotel B)
Solve for y, when $x = 4$	Solve for y, when $x = 4$
$y = 500x + 12{,}000$	$y = 1000x + 10{,}500$
$y = 500(4) + 12{,}000$	$y = 1000(4) + 10{,}500$
$y = 2{,}000 + 12{,}000$	$y = 4{,}000 + 10{,}500$
$\boldsymbol{y}$ $\mathbf{14,000}$	$\boldsymbol{y}$ $\mathbf{14,500}$

At $x = 4$, the cost of Hotel A is lower than the cost of Hotel B.

This means that Hotel B is recommended if the extended number of hours is less than 3 hours, both hotels A & B can be recommended if the extended number of hours is exactly 3 hours, and Hotel A is recommended if the extended number of hours is more than 3 hours.



What's More

Activity 1: Substitute!

Directions: Solve each system of equations by substitution. Check your solutions.

1. $\begin{cases} y = 2x \\ x + y = 6 \end{cases}$ 2. $\begin{cases} 5x + 10y = 3 \\ x = -\frac{1}{2}y \end{cases}$ 3. $\begin{cases} y - x = 3x + 2 \\ 2x + 2y = 14 - y \end{cases}$

Activity 2: Who Will Be Eliminated?

Directions: Identify the terms that can be eliminated. If elimination by addition or subtraction cannot be directly performed, state first the number or numbers that should be multiplied to one or both equations (see example). Then solve each system by elimination. Check your solutions.

$\boldsymbol{Example}$: $\begin{cases} \mathbf{3x - 4y = -7} \\ \mathbf{2x + y = 10} \end{cases}$ ⇒ Multiply first equation 2 by 4, then eliminate $(-4y)$ and $4y$.

1. $\begin{cases} \boldsymbol{x+y} = \boldsymbol{-1} \\ \boldsymbol{x-y=3} \end{cases} \Rightarrow \boxed{}$

2. $\begin{cases} \boldsymbol{x+y} = \boldsymbol{5} \\ \boldsymbol{x+2y=8} \end{cases} \Rightarrow \boxed{}$

3. $\begin{array}{c} x+y=1 \\ 2x-2y=4 \end{array} \Rightarrow \boxed{}$

4. $\begin{array}{c} 4x+4y=6 \\ 5x-2y=4 \end{array} \Rightarrow \boxed{}$

Questions:

1. How did you identify the terms to be eliminated?
2. What have you noticed with the numerical coefficients of the variables of the terms to be eliminated?
3. What operation should be used to eliminate a variable in item 1? in item 2?
4. Can any of the variable be directly eliminated in items 3, and 4? Why or Why not?
5. Is there a need to find equivalent equations in 3, and 4 to eliminate a variable? What are these new equations?

Activity 3: Am I consistent or Inconsistent?

Directions: Solve each system of equations by substitution or elimination. Determine whether the system is consistent and independent, consistent and dependent, or inconsistent.

1. $\begin{cases} x-y=3 \\ -3x+3y=1 \end{cases}$

2. $\begin{cases} 2x+3y=-5 \\ -6x-6y=15 \end{cases}$

3. $\begin{cases} y=\frac{1}{2}x \\ 2x+y=2 \end{cases}$

Questions:

1. Which system of equations is consistent and independent? consistent and dependent? inconsistent?
2. How will you describe the solution to each system of equations?

Activity 4: Following Protocols

Directions: Solve the problem below by illustrating the process of finding solution. Write your answer on a separate sheet of paper.

Matt and Ming are selling fruit for a school fundraiser. Customers can buy small and large boxes of oranges. Matt sells 3 small boxes and 14 large boxes of oranges for a total of Php 203.00. Ming sells 11 small boxes of oranges and 11 large boxes of oranges for a total of Php 220.00. Find the cost of a small box and large box of oranges.

Step 1. Understand the problem	Let x __________ Let y __________
Step 2. Devise a plan (translate)	Equation 1:______________ Equation 2:______________

Step 3. Carry out the plan (solve).	Solution:
Step 4. Look back (Check and interpret)	Check:

Activity 5: Problems Solved!

Directions: Read each problem carefully and solve as required. Then answer the questions that follow. Use a separate sheet of paper.

A. The Number Game

The sum of two numbers is 90. The larger number is 14 more than 3 times the smaller number. Find the numbers.

Questions:

1. What equations can be formed to determine the two numbers?
2. What method of solving systems of linear equation in two variables can best be applied to solve this problem?
3. What are the two numbers?

B. Chocolate Desires

White chocolate costs Php 20.00 per bar, and dark chocolate costs Php 25.00 per bar. If Janine bought 15 bars of chocolate for Php 340, how many bars of dark chocolate did she buy?

Questions:

1. What two equations can be formed to represent the number of chocolate bars?
2. What method of solving systems of linear equations in two variables can best be applied to solve this problem? Why do you think the method you chose is appropriate to solve this type of problem?
3. How many bars of dark chocolate did Janine buy?

C. Bonding, Bonding…

It's vacation time of the year and Luigi's family agreed to go to a famous beach resort in their province. Upon entering the resort, they were asked to pay tickets which cost $Php200.00$ for children (5 to 12 years old) and $Php450.00$ for adults. If the resort were able to sell 250 pieces of beach ticket amounting to $Php76,000.00$, how many children and adults were in the beach?

Questions:

1. What are the two equations that can be used to find the number of children and adults in the beach?
2. How many children and adults were in the beach?

What I Have Learned

A. You Complete Me!

Direction: Complete each statement below.

In this lesson, I learned the steps in solving a system of linear equations in two variables using substitution method.

First, I ____

After that, I ____

Then, ____

Finally, ____

When I have completed these steps, I have shown that ________________________

____.

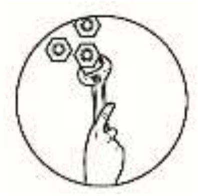
B. Put Me in My Right Place!

Directions: Fill in the blank spaces of the paragraph below with correct word/s or expression/s which you can choose from the box. Word/s or expression/s in the box may be used more than once.

add	eliminate	solution	2	variable	subtract
adding	elimination	(2,0)	0	(0,2)	multiply

Aside from substitution method, systems of linear equations in two variables can also be solved by _________ method. In the _________ method, I can either _________ or _________ the equations to solve an equation for one _________. When the coefficients of one variable are opposites, I can _________the equations to _________a variable and when the coefficients of one variable are equal, I can _________the equations to _________ a variable.

In the system $\begin{cases} 3x+2y=\ 6 \\ 5x-2y=10 \end{cases}$, y can be eliminated by _________ the two equations. After _______the two equations, the result would now be equal to $8x=16$. This means that $x\ =$_________. Knowing that $x\ =$ ___, I can now solve for the value of y by using any of the equations. Hence, the value of $y=$_________. The value of x and y in the ordered pair _________ can be now substituted to both equations to check whether the ordered pair satisfies both equations. Both equations are true after substituting the obtained values of *x* and *y*. This means that the ordered pair (_____, ______), is a ____________ to the system.



What I Can Do

Activity 1: Let's Go Shopping!

Directions: Read the problem below and answer the questions that follow. Show your solutions when necessary. Use a separate sheet of paper.

You and your friends went to mall with Php1,000.00 to spend for shopping. In one of the stalls for ready-to-wear items, you found out that jeans and blouses are on sale. One brand of jeans cost Php300.00 each while all blouses in that same brand were sold at Php100.00. You wanted to buy a total 6 items. Wouldn't it be clever to find how many jeans and blouses can your buy with your money?

Questions:

1. Find the equations representing the given in the situations.
2. How many jeans and blouses can you buy with your money?
3. How do you classify the system of equations? Is it consistent and independent, consistent and dependent, or inconsistent?

Activity 2: Let's Drink!

Directions: Read the problem and solve. Then, answer the questions that follow. Show your solutions.

Drinking adequate amount of water may boost learning. It will help you concentrate better in school. Melba, one of your classmates wanted to track how much water she consumed each day. She has one small and one large reusable bottle. Yesterday, Melba drank 5 small bottles of water and 1 large bottle of water for a total of 180 ounces. The day before that, she drank 4 small bottles of water and 1 large bottle of water for a total of 155 ounces. How much does each water bottle hold?

Questions:

1. Determine the equations that would represent the situation.
2. Which method of solving systems of linear equations in two variables is best to use to determine the number of ounces each bottle hold?
3. How much does each water bottle hold?
4. Do you think Melba drinks enough water a day? Why?

Assessment

Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which method is best to use when the numerical coefficients of the variables are either $1\ or\ -1$?
   A. algebraic
   B. elimination
   C. graphical
   D. substitution

2. Which method is best to apply to solve the system $\begin{cases} y = 3x - 2 \\ 2x + y = 8 \end{cases}$?
   A. algebraic
   B. elimination
   C. graphical
   D. substitution

3. Which of the following equations can best be solved using elimination by addition?
   A. $\begin{cases} 2x + y = 10 \\ 2x + y = 8 \end{cases}$
   B. $\begin{cases} x + y = 5 \\ x - y = 7 \end{cases}$
   C. $\begin{cases} x + y = 10 \\ 2x + y = 8 \end{cases}$
   D. $\begin{cases} x - 3y = 5 \\ 3 + x = -2y \end{cases}$

4. The system of linear equation $\begin{cases} y = 2x - 9 \\ x + 3y = 8 \end{cases}$ is solved by substitution. After the initial substitution in the second equation, which of the following is the resulting expanded equation?
   A. $x + 3y - 9 = 8$
   B. $x + 3(2x) - 9 = 8$
   C. $x + 3(2x) - 9 = 8$
   D. $x + 3(2x - 9) = 8$

5. The first thing to do when solving systems of linear equation by elimination is rewriting equations into the standard form $Ax + By = C$. Which of the following is the resulting system of equations when $\begin{cases} 3x + 2y + 1 = 5 \\ 3\,(x - 1) = -2y - 4 \end{cases}$ is written in standard form?
   A. $\begin{cases} 3x + 2y = 6 \\ 3x + 2y = 7 \end{cases}$
   B. $\begin{cases} 3x + 2y = 4 \\ 3x + 2y = 7 \end{cases}$
   C. $\begin{cases} 3x + 2y = 4 \\ 3x + 2y = -1 \end{cases}$
   D. $\begin{cases} 3x + 2y = 6 \\ 3x - 2y = -3 \end{cases}$

6. The system of linear equation $\begin{cases} 3x + 12y = -12 \\ x - 3y = 10 \end{cases}$ is to be solved using elimination method. What should be the first step to solve this system?
   A. Add the two equations.
   B. Subtract the equations.
   C. Multiply the first equation by -4
   D. Multiply the second equation by 4

7. Given $\begin{cases} 2x - 3y = 10 \\ 3x - 2y = -5 \end{cases}$, which of the following is its equivalent system with same $x-$coefficients?

A. $\begin{cases} 6x - 9y = 10 \\ 6x - 2y = -10 \end{cases}$

C. $\begin{matrix} 6x - 9y = 30 \\ 6x - 4y = -10 \end{matrix}$

B. $\begin{matrix} 6x - 3y = 10 \\ 6x - 2y = -5 \end{matrix}$

D. $\begin{matrix} 6x - 9y = 30 \\ 6x - 4y = 10 \end{matrix}$

8. Given $\begin{cases} 2x - 3y = 10 \\ 3x + 2y = -5 \end{cases}$, which of the following is its equivalent system with opposite $y - coefficients$?

A. $\begin{cases} -4x + 6y = -20 \\ -9x - 6y = -15 \end{cases}$

C. $\begin{cases} 4x - 6y = 20 \\ 9x + 6y \ = -15 \end{cases}$

B. $\begin{cases} 4x + 6y = 20 \\ -9x - 6y = -15 \end{cases}$

D. $\begin{cases} 4x + 6y = 20 \\ -9x - 6y = -15 \end{cases}$

9. In three more years, Miguel's grandfather will be six times as old as Miguel was last year. When Miguel's present age is added to his grandfather's present age, the total is 68. How old is Miguel now?

A. 9
C. 11
B. 10
D. 12

10. The sum of two numbers is 15. If twice the first number is added to thrice the second number their sum would be 35. What are the numbers?

A. 7 and 8
C. 10 and 5
B. 9 and 6
D. 12 and 3

11. A total of 315 Grade 8 students participated in a community outreach program organized by the local government. Some students rode in vans which hold 9 passengers each and some students rode in buses which hold 22 passengers each. How many of each type of vehicle did they use if there were 22 vehicles in total?

A. 9 vans and 13 buses
C. 15 vans and 7 buses
B. 13 vans and 9 buses
D. 7 vans and 15 buses

12. 3 bags and 2 pairs of shoes cost Php1, 500.00 while 5 bags and 8 pairs of shoes cost Php4 950.00. What is the cost of each bag and a pair of shoes?

A. Each bag cost Php250.00 and each pair of shoes cost Php425.00.
B. Each bag cost Php275.00 and each pair of shoes cost Php400.00.
C. Each bag cost Php200.00 and each pair of shoes cost Php475.00.
D. Each bag cost Php150.00 and each pair of shoes cost Php525.00.

13. A farmyard has dogs and chickens. The owner said that his dogs and chickens had a total of 148 legs and 60 heads. How many dogs and chickens were in the farmyard?

A. 22 dogs and 38 chickens
B. 38 dogs and 22 chicken
C. 14 dogs and 46 chickens
D. 46 dogs and 14 chickens

14. Melba says that the system $\begin{cases} -x + 4y = 6 \\ x + y = -3 \end{cases}$ has exactly one solution. Which of the following reasons would support her statement?

I. An ordered pair $\left(-\frac{18}{5}, \frac{3}{5}\right)$ satisfies both equations.
II. An ordered pair $\left(\frac{18}{5}, -\frac{3}{5}\right)$ satisfies both equations.
III. The system becomes a true statement after eliminating a variable.
IV. The system becomes a FALSE statement after eliminating a variable.

A. I only
B. I and II
C. II and III
D. IV only

15. Trish was asked by her Math teacher to solve the system $\begin{cases} x = 2y + 3 \\ 2x + 3y = -3 \end{cases}$. She decided to use substitution method to solve the system. Which of the following statements justify her choice of method?

I. It is always the recommended method for systems with one solution.
II. It is recommended because one of the equations is not in standard form.
III. Substitution should be used since one of the equations is already solved in terms of one variable.

A. I
B. II
C. III
D. I and II

Additional Activities

Let us Explore Further!

Directions: Answer each question as directed. Solve when necessary. Use a separate sheet of paper.

1. Write the equivalent equations without fractions for each equation in the system. Solve the system.

$$\begin{cases} \dfrac{5x-2}{4}+\dfrac{1}{2}=\dfrac{3y+2}{2} \\ \dfrac{7y+3}{3}=\dfrac{x}{2}+\dfrac{7}{3} \end{cases}$$

2. Is it possible to use substitution or elimination to solve a system of linear equations in two variables if one equation represents a vertical line and the other equation represents a horizontal line? Show and explain your answer.

3. Using a Venn Diagram, compare and contrast Graphing, Elimination, and Substitution Methods in solving systems of linear equations in two variables.

Answer Key

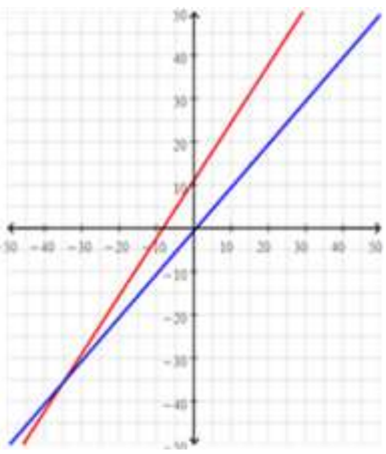
What I Know

1. B
2. C
3. B
4. C
5. A
6. B
7. B
8. C
9. B
10. A
11. A
12. D
13. A
14. C
15. D

What's In

Original Equation	Transformed Equation
1. $x + y = 2$	$x = -y + 2$
2. $x - y = \frac{2}{3}$	$x = y + \frac{2}{3}$
3. $3x + 2y = 6$	$x = -\frac{2}{3}y + 2$
4. $\frac{1}{4}x + y = 2$	$y = -\frac{1}{4}x + 2$
5. $4x - 3y = -33$	$y = \frac{4}{3}x + 11$

WHAT'S NEW



Point of Intersection: $(-35, \frac{-107}{3})$

What's More

Activity 1: SUBSTITUTE

1. (2, 4) 2. $\left(-\frac{1}{5}, \frac{2}{5}\right)$ 3. $\left(\frac{4}{7}, \frac{30}{7}\right)$

Activity 2: Who will be eliminated?

1. eliminate $(+y) and\ (-y)$
Solution: (1, -2)
2. multiply first equation by -1 and then eliminate $(-x) and\ (x)$.
Solution: (2, 3)
3. multiply first equation by 2 and then eliminate $(2y) and\ (-2y)$
Solution: $\left(\frac{3}{2}, -\frac{1}{2}\right)$
4. multiply second equation by 2 and then eliminate $4y\ and (-4y)$
Solution: $\left(1, \frac{1}{2}\right)$

Activity 3: Am I consistent or Inconsistent?

1. NO solution
2. $\left(-\frac{5}{2}, 0\right)$ 3. $\left(\frac{4}{5}, \frac{2}{5}\right)$

QUESTIONS:

1. System 1: Inconsistent
System 2: consistent and Independent
System 3: Consistent and Independent
2. Aside from system 1 that doesn't have a solution; the solutions of system 2 and 3 both satisfy the equations correspondingly.

Activity 4: Following Protocols

Step 1: let x be small box
Let y be large box

Step 2:
Equation 1: $3x + 14y = 203$
Equation 2: $11x + 11y = 220$

Step 3: Solution:

$$-11(3x + 14y = 203)$$
$$3(11x + 11y = 220)$$

$$-33x - 154y = -2{,}233$$
$$\underline{33x + 33y = 660}$$
$$\frac{-121y}{-121} = \frac{-1{,}573}{-121}$$
$$\boldsymbol{y = 13}$$

Substitute the value of y to the 1st equation.

$$3x + 14y = 203$$
$$3x + 14(13) = 203$$
$$3x + 182 = 203$$
$$3x = 203 - 182$$
$$3x = 21$$
$$\boldsymbol{x = 7}$$

Step 4: Check and Interpret.

Equation 1: 3x + 14y = 203
3(7) +14(13) = 203
21+182 = 203
203 = 203

Equation 2: 11x + 11y = 220
11(7) + 11(7) = 220
77 + 143 = 220
220 = 220

Therefore; small box costs Php7.00 and large box costs Php13.00

Activity 5: Problems Solved!

A. The Number Game

1. $x + y = 90\ and\ x = 3y + 14$
2. Substitution method
3. 71 and 19

B. Chocolate Desires

1. $x + y = 15$ and $20x + 25y = 340$
2. Answers may vary.
3. 8 bars of dark chocolate

C. Bonding, Bonding

1. $x + y = 250\ and\ 200x + 450y = 76{,}000$
2. 146 children and 104 adults

WHAT HAVE I LEARNED

A. You Complete Me!

Answers may vary.

B. Put Me in My Right Place!

Aside from substitution method, systems of linear equations in two variables can also be solved by elimination method. In the elimination method, I can either add or subtract the equations to solve an equation for one variable. When the coefficients of one variable are opposites, I can add the equations to eliminate a variable and when the coefficients of one variable are equal, I can subtract the equations to eliminate a variable.

In the system $\begin{cases} 3x + 2y = 6 \\ 5x - 2y = 10 \end{cases}$, y can be eliminated by adding the two equations. After adding the two equations, the result would now be equal to $8x = 16$. This means that $x = 2$. Knowing that $x = 2$, I can now solve for the value of y by using any of the equations. Hence, the value of $y = 0$. The value of x and y in the ordered pair $(2, 0)$ can be now substituted to both equations to check whether the ordered pair satisfies both equations. Both equations are true after substituting the obtained values of x and y. This means that the ordered pair $(2, 0)$, is a solution to the system.

What I Can Do

Activity 1: Let's Go Shopping!

1. let x be the blouse
   Let y be the jeans

$$x + y = 6$$
$$100x + 300y = 1,000$$

2. 4 blouses and 2 jeans
3. consistent and independent

Activity 2: Let's Drink!

1. let x be the water small bottle holds
   let y be the water large bottle holds

$$5x + y = 180$$
$$4x + y = 155$$

2. Elimination Method
3. Small bottle holds 25 ounces of water while large bottle holds 55 ounces of water
4. Yes. Because a person should usually drink 8- 10glass of water a day and Melba had been drinking water up to 180 ounces a day which is more than 8 glasses of water.

Assessment

1. B
2. D
3. B
4. D
5. C
6. D
7. C
8. C
9. C
10. C
11. B
12. D
13. C
14. A
15. C

Additional Activities:

1. $\begin{cases} 5x - 6y = 4 \\ 3x - 14y = -8 \end{cases}$; $(2,1)$

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For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph